Streaming Algorithms: Data without a disk

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CSE545
Spring 2020

Motivation

One often does not know when a set of data will end.

- Can not store
- Not practical to access repeatedly
- Rapidly arriving
- Does not make sense to ever “insert” into a database

Can not fit on disk but would like to generalize / summarize the data?

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Examples: Google search queries

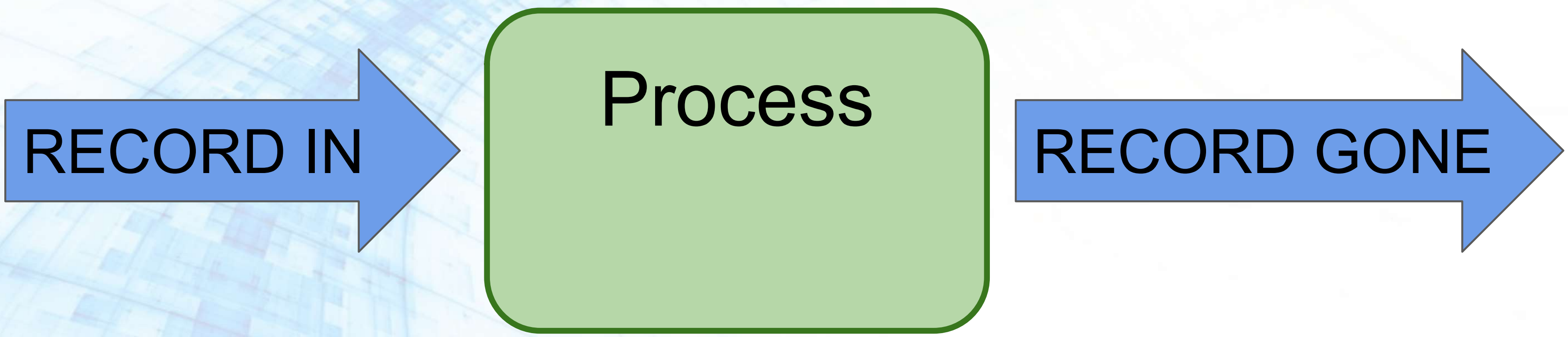
Satellite imagery data

Text Messages, Status updates

Click Streams

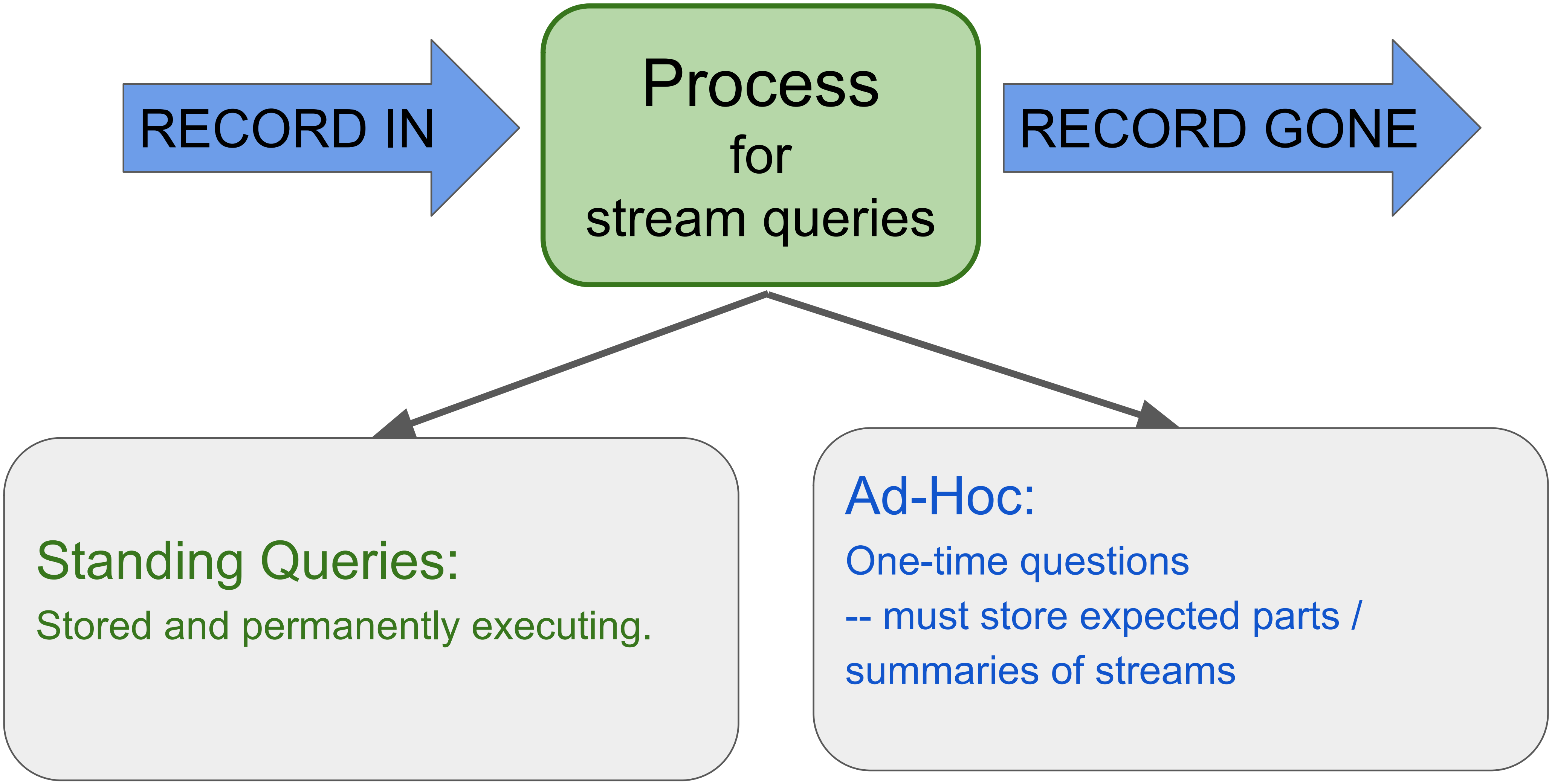
Motivation

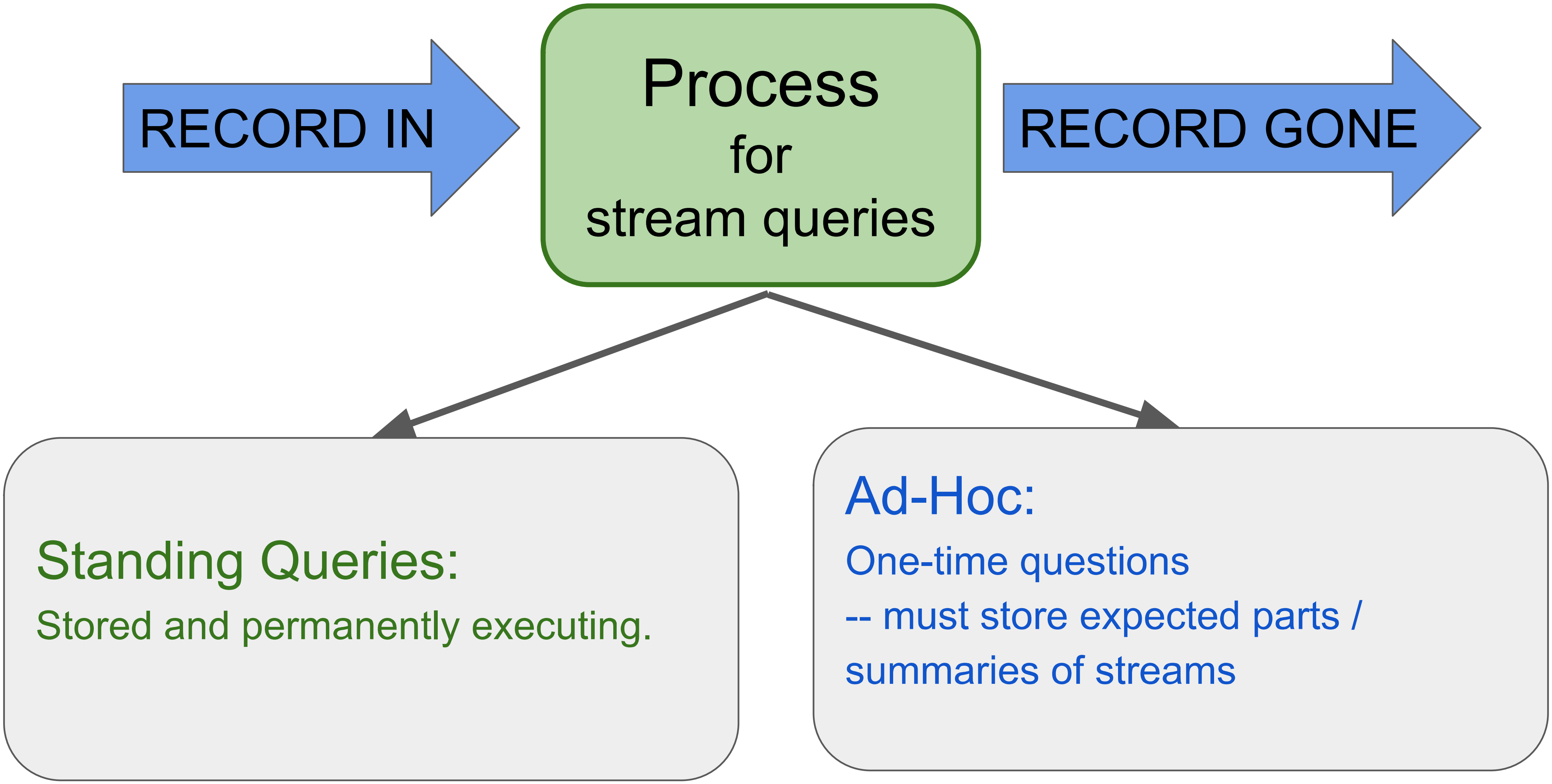
Often translate into $O(N)$ or strictly N algorithms.



Streaming Topics

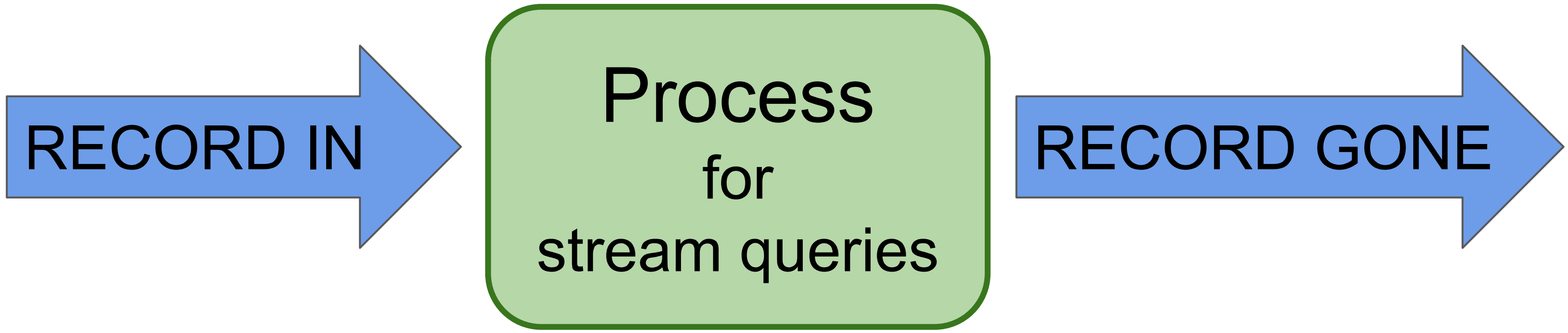
- General Stream Processing Model
- Sampling
- Filtering data according to a criteria
- Counting Distinct Elements


RECORD IN
Process
for
stream queries
RECORD GONE
Standing Queries:
Stored and permanently executing.
Ad-Hoc:
One-time questions
-- must store expected parts /
summaries of streams



E.g. How would you handle:

What is the mean of values seen so far?



Important difference from typical database management:

- Input is not controlled by system staff.
- Input timing/rate is often unknown, controlled by users.

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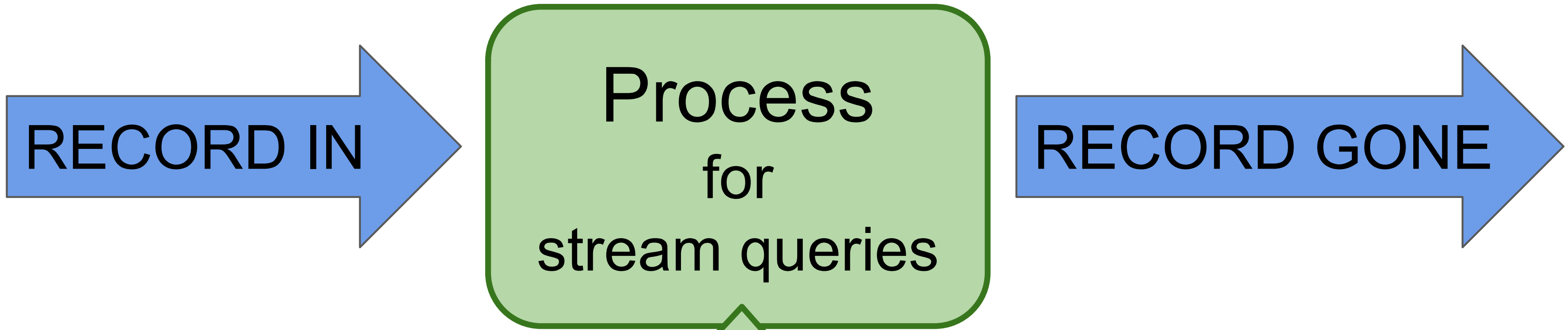

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- Input is n...
- Input timing/rate is ... controlled by users.

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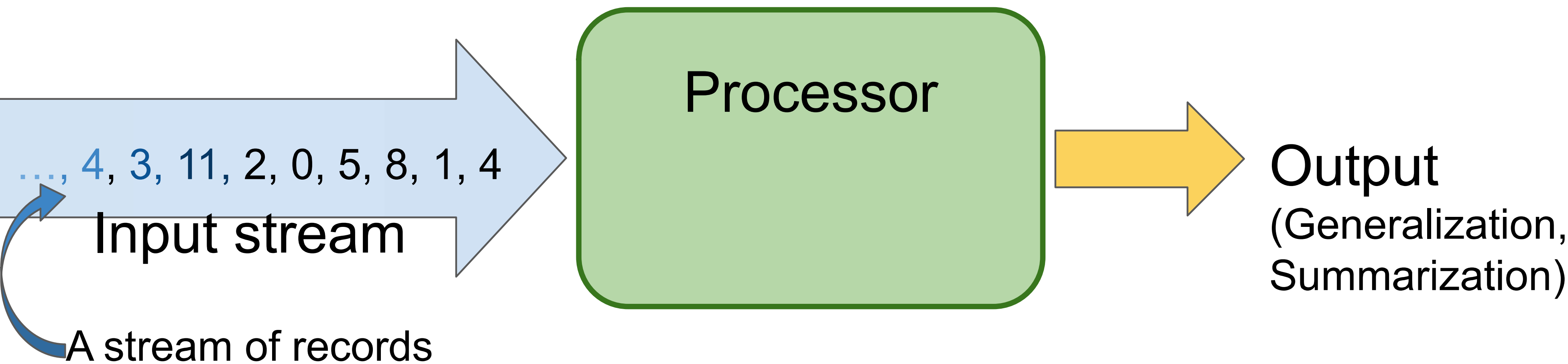
General Stream Processing Model

(Leskovec et al., 2014)



A stream of records
(also often referred to as "elements" or "tuples")
Theoretically, could be anything! *search queries, numbers, bits, image files, ...*

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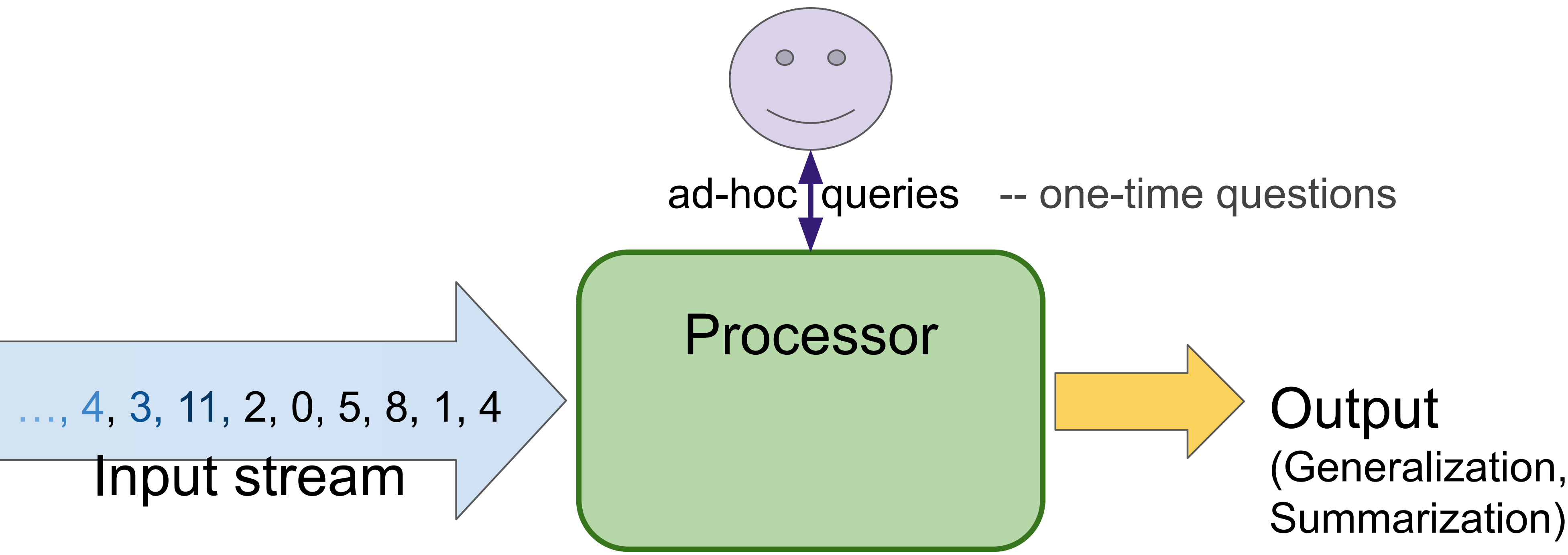

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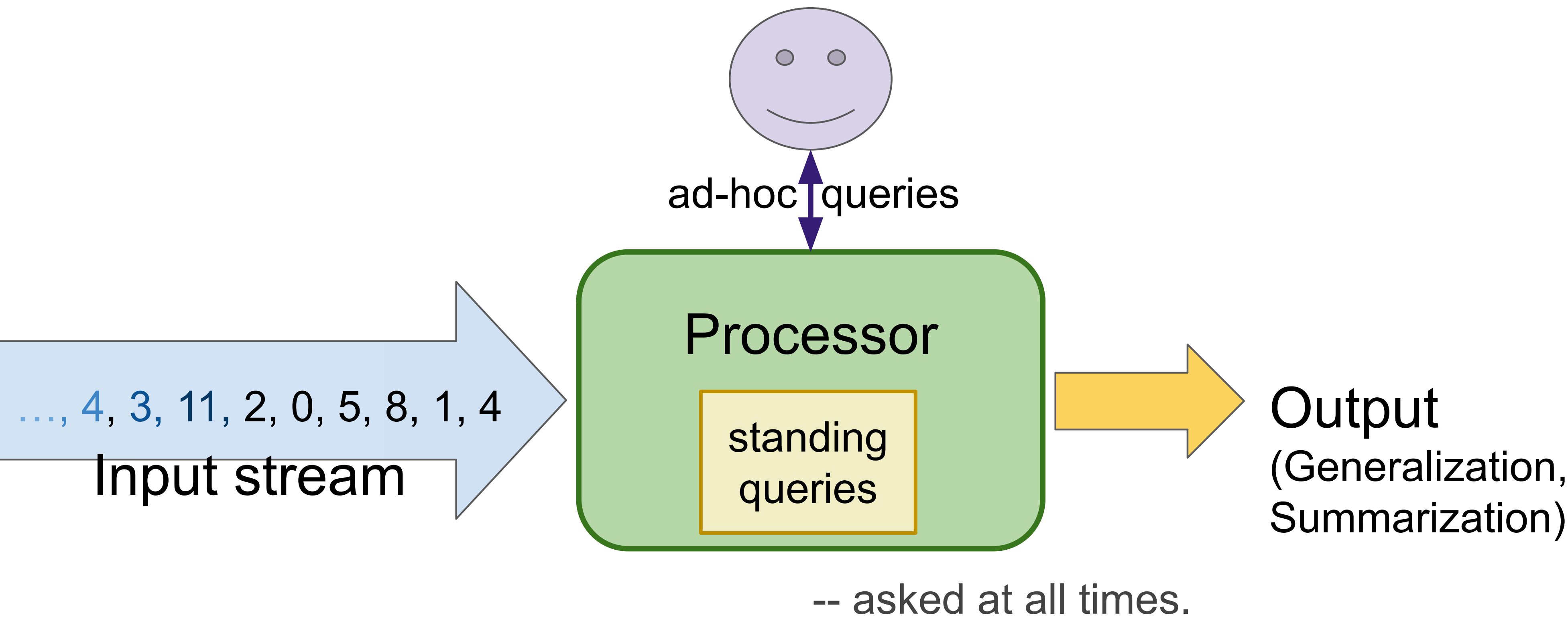

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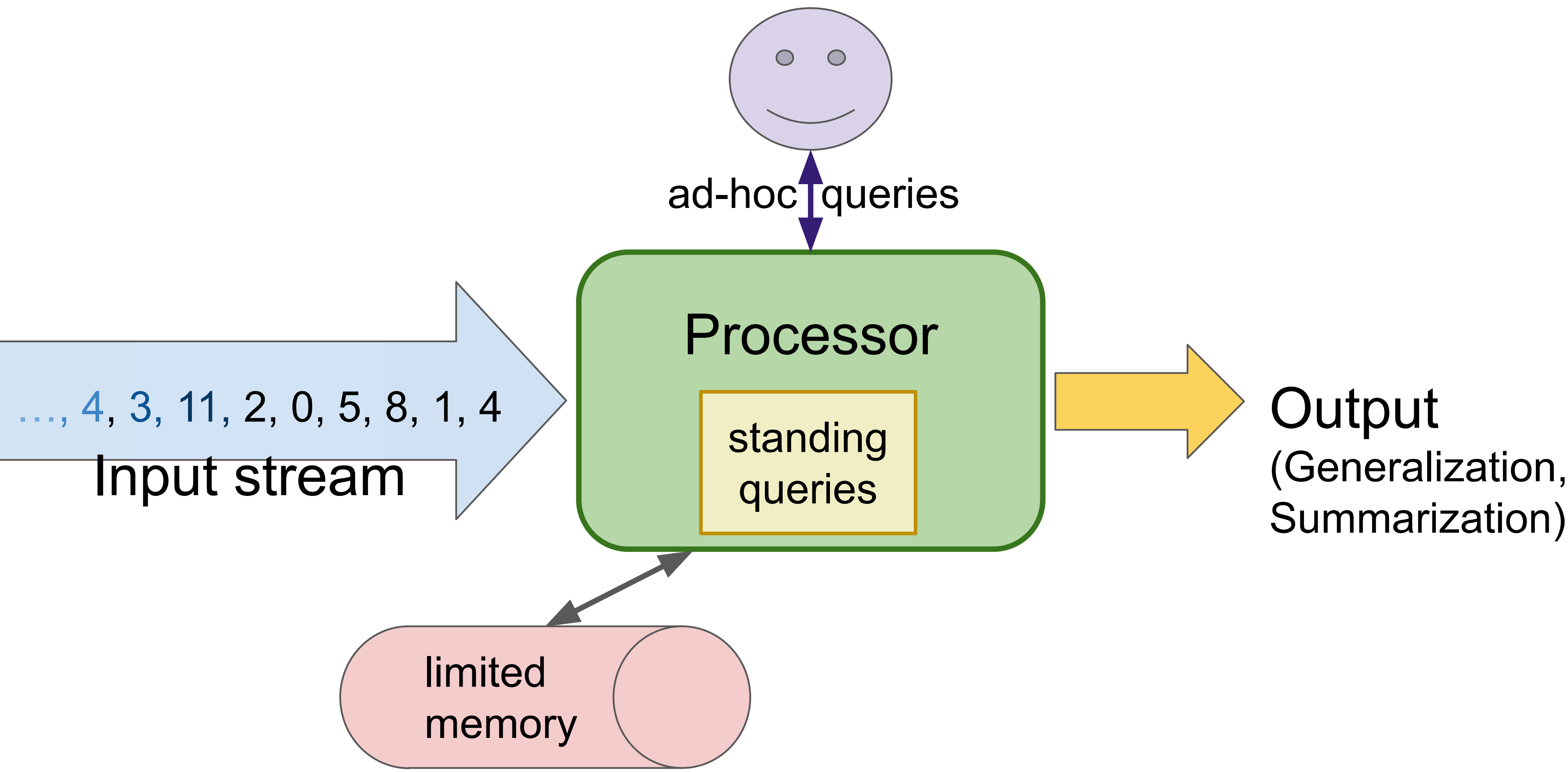

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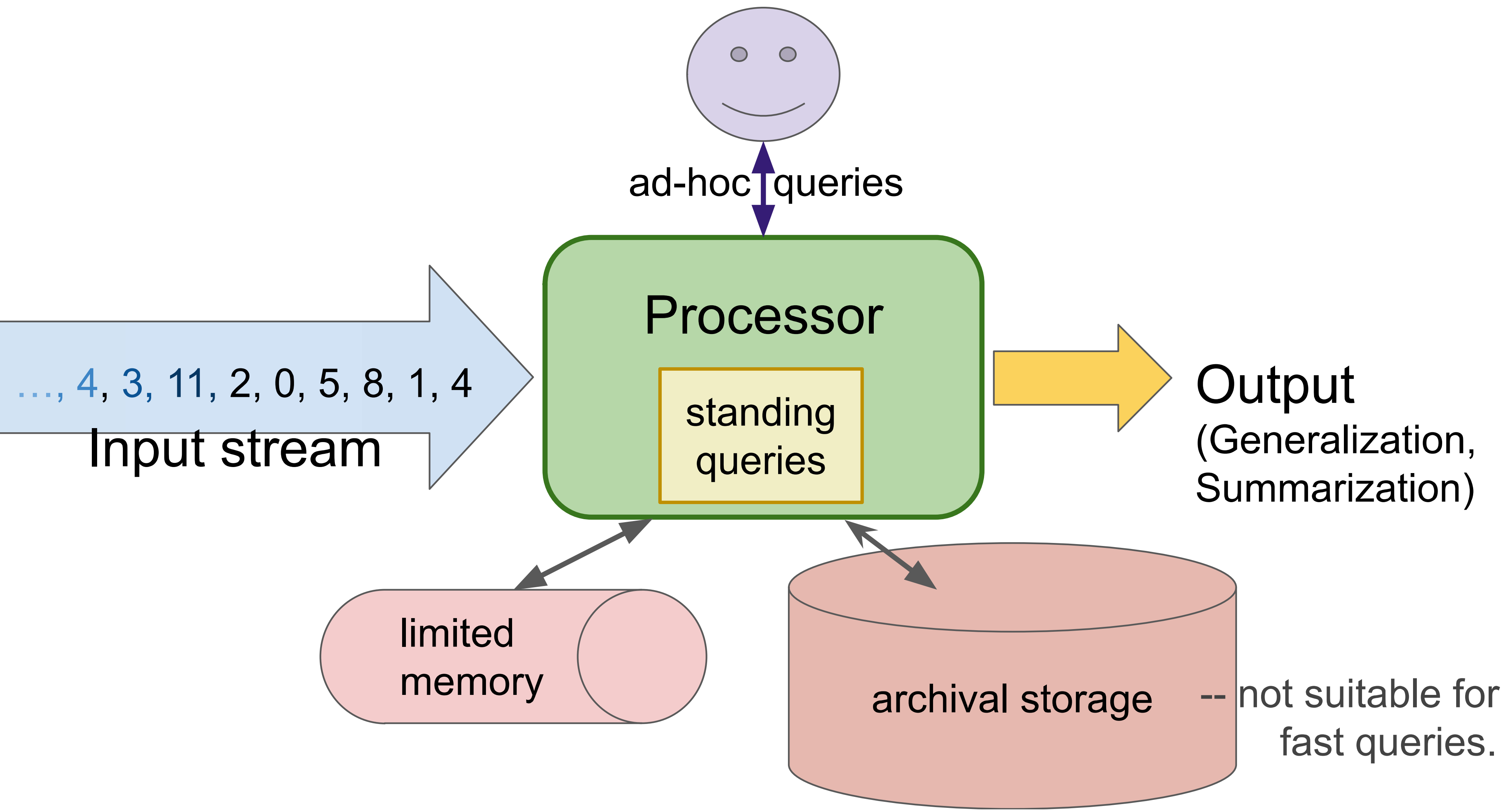

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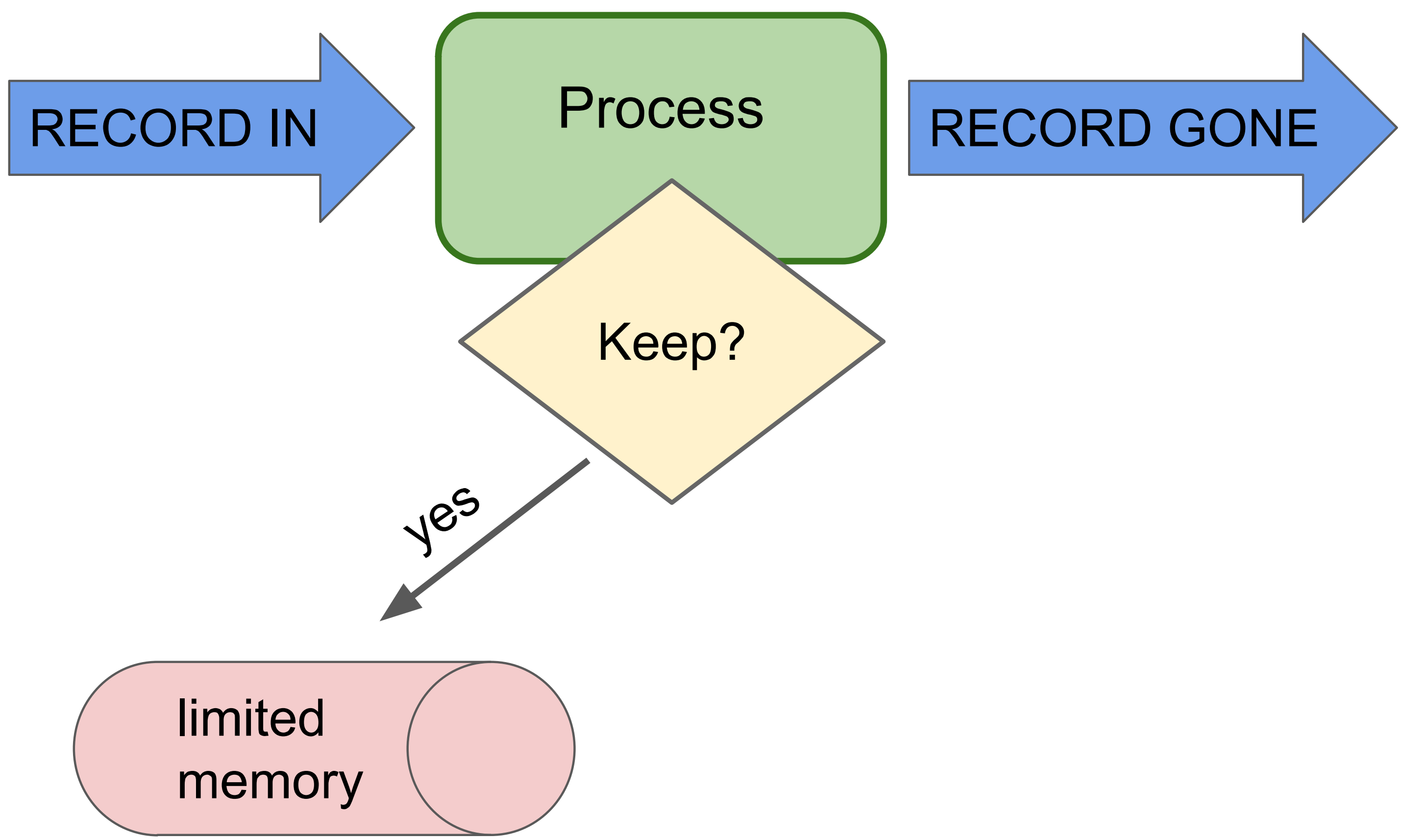

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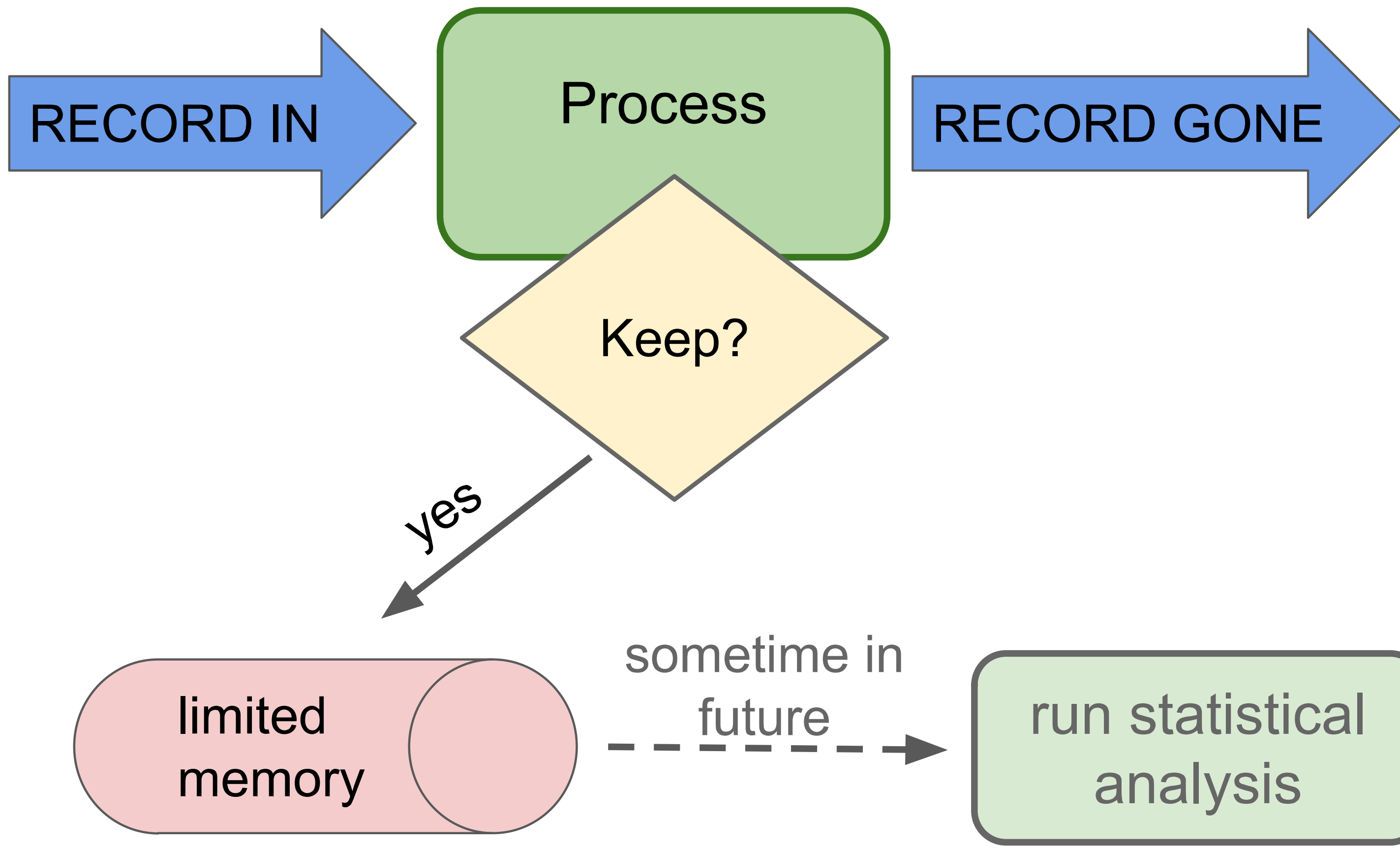

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if random() <= .05: #keep: true 5% of the time
    memory.write(record)
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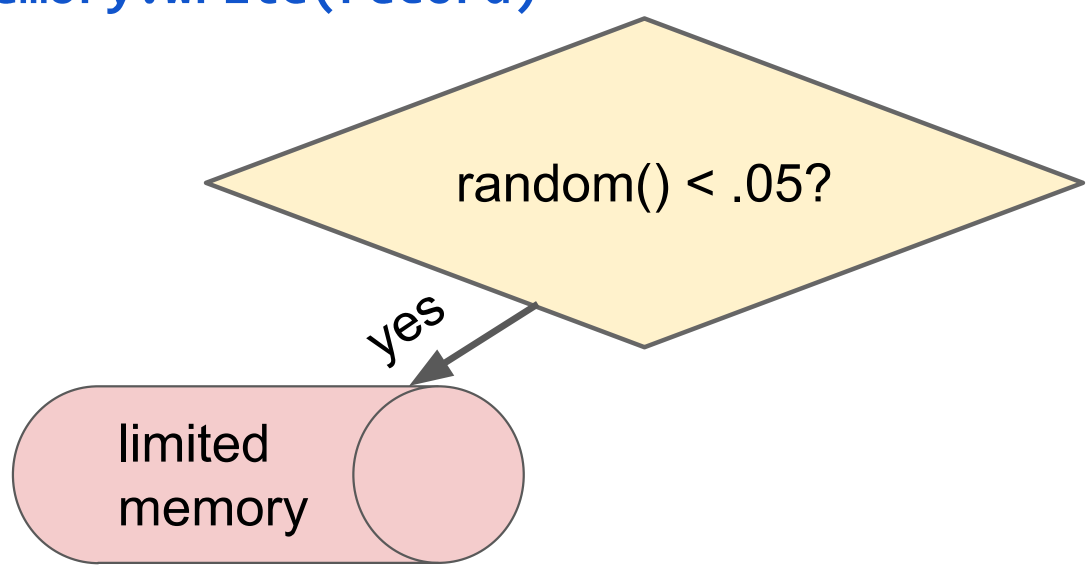

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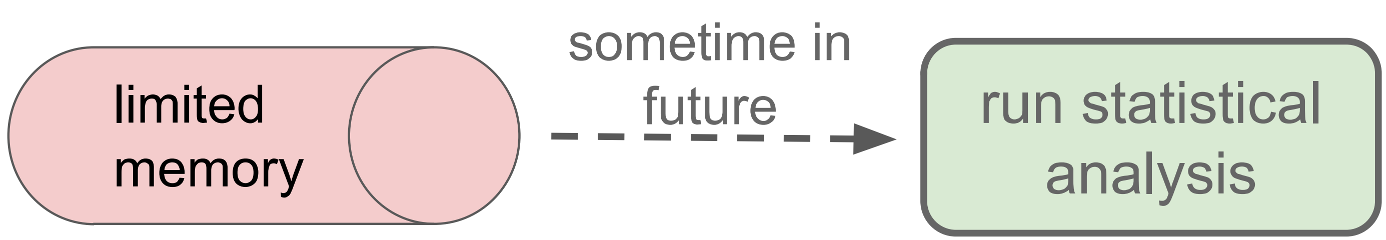

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Solution: hash into *N* = *1/perc* buckets; designate 1 bucket as “keep”.

```
if hash(record[‘user_id’]) == 1: #keep
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only need to store hash functions; may be part of standing query

Filtering Data

Filtering: Select elements with property x

Example: 40B safe email addresses for spam filter

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The Bloom Filter (approximates; allows *false positives but not false negatives*)

Given:

$|S|$ keys to filter; will be mapped to $|B|$ bits

hashes = $h_1, h_2, \ldots, h_k$ independent hash functions

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  ... #usually embedded in other code
  while key x arrives next in stream #filter:
    if B[h_i(x)] == 1 for all i in hashes:
      do as if x is in S
    else: do as if x not in S
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What is the probability of a *false positive (FP)*?

Q: What fraction of |B| are 1s?

(Leskovec et al., 2014)

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A: Analogy:

Throw |S| * k darts at n targets.

1 dart: $1/n$

d darts: $(1 - 1/n)^d$ = prob of 0

= $e^{-d/n}$ are **0s**

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$= e^{-1}$
for large n

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thus, $(1 - e^{-d/n})$ are **1s**

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probability all *k* being 1?

$(1 - e^{-(|S|*k)/n})^k$

Note: Can expand S as stream continues as long as |B| has room (e.g. adding verified email addresses)

(Leskovec et al., 2014)

Counting Moments

Moments:

- Suppose m_i is the count of distinct element i in the data
- The kth moment of the stream is $\sum_{i \in \mathrm{Set}} m_i^k$

- 0th moment: count of distinct elements
- 1st moment: length of stream
- 2nd moment: sum of squares
  (measures *uneveness;* related to variance)

Counting Moments

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Trivial: just increment
a counter

- 0th momen[illegible] of distinct elements
- **1st moment: length of stream**
- 2nd moment: sum of squares
  (measures *uneveness;* related to variance)

Counting Momen

0th moment

Applications

Counting…

- distinct words in large document.
- users that visit a site.
- unique queries to Alexa.

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Counting Momen

Applications
Counting…
distinct words in large document.
distinct websites (URLs).
users that visit a site.
unique queries to Alexa.

0th moment
One Solution: Just keep a set (hashmap, dictionary, heap)

Problem: Can't maintain that many in memory; disk storage is too slow

- **0th moment: count of distinct elements**
- 1st moment: length of stream
- 2nd moment: sum of squares
  (measures *uneveness;* related to variance)

Counting Moments

0th moment
Streaming Solution: Flajolet-Martin Algorithm
General idea:

- n -- suspected total number of elements observed
- pick a hash, *h*, to map each element to $\log_2 n$ bits (buckets)

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  (measures *uneveness;* related to variance)

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--------------------------------
R = 0 #potential max number of zeros at tail
for each stream element, e:
    r(e) = trailZeros(h(e)) #num of trailing 0s from h(e)
    R = r(e) if r[e] > R

estimated_distinct_elements = 2^R
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estimated_distinct_elements = 2^R # m
```

Mathematical Intuition

P(trailZeros(*h*(*e*)) >= *i*) = 2^{-i}

 # P(h(e) == __0) = .5; P(h(e) == __00) = .25; …

P(trailZeros(*h*(*e*)) < *i*) = $1 - 2^{-i}$

for m elements: = $(1 - 2^{-i})^m$

P(one *e* has tailZeros > i) = $1 - (1 - 2^{-i})^m$

$\approx 1 - e^{-m2^{\wedge}-i}$

If $2^R >> m$, then $1 - (1 - 2^{-i})^m \approx 0$

If $2^R << m$, then $1 - (1 - 2^{-i})^m \approx 1$

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Problem:

Unstable in practice.

Solution:

Multiple hash functions

but how to combine?

- 2nd moment: sum of squares
  (measures *uneveness;* related to variance)

0th moment

Streaming Solution: Flajolet-Martin Algorithm

General idea:

n -- suspected total number of elements

pick a hash, *h,* to map each element to l

```
Rs = list()
for h in hashes:
    R = 0 #potential max number of zeros at tail
    for each stream element, e:
        r(e) = trailZeros(h(e) #num of trailing 0s from h(e)
        R = r(e) if r[e] > R
    Rs.append(2^R)

groupRs = [Rs[i:i+log n] for i in range(0, len(Rs), log n)]

estimated_distinct_elements = median(map(mean, groupRs))
```

Problem:

Unstable in practice.

Solution: Multiple hash functions

1. Partition into groups of size log n
2. Take mean in groups
3. Take median of group means

0th moment

Streaming Solution: Flajolet-Martin Algorithm

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A good approach anytime one has many “low resolution” estimates of a true value.

Counting Moments

2nd moment
Streaming Solution: Alon-Matias-Szegedy Algorithm

(Exercise; Out of Scope; see in MMDS)

- 0th moment: count of distinct elements
- 1st moment: length of stream
- **2nd moment: sum of squares (measures *uneveness* related to variance)**